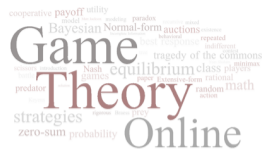


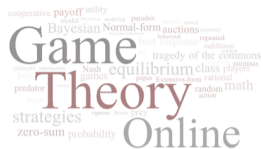
Analyzing Bayesian Games

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Bayesian (Nash) Equilibrium



Bayesian (Nash) Equilibrium



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Bayesian (Nash) Equilibrium



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- **Mixed strategy:** $s_i : \Theta_i \mapsto \Pi(A_i)$
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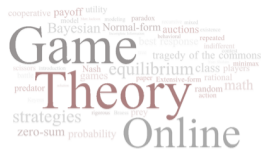
Interim expected utility

- Given a Bayesian game (N, A, Θ, p, u) with finite sets of players, actions, and types, i 's *interim expected utility* with respect to type θ_i and a mixed strategy profile s is

$$EU_i(s|\theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} p(\theta_{-i}|\theta_i) \sum_{a \in A} \left(\prod_{j \in N} s_j(a_j|\theta_j) \right) u_i(a, \theta_i, \theta_{-i}).$$



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- i 's **ex ante expected utility** with respect to a mixed strategy profile s is

$$EU_i(s) = \sum_{\theta_i \in \Theta_i} p(\theta_i) EU_i(s|\theta_i).$$

Bayesian Equilibrium or Bayes-Nash equilibrium

A **Bayesian equilibrium** is a mixed strategy profile s that satisfies

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for each i and $\theta_i \in \Theta_i$.



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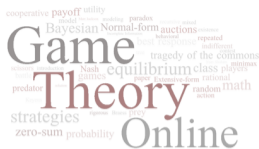
If $p(\theta_i) > 0$ for all $\theta_i \in \Theta_i$, then this is equivalent to requiring that

$$s_i \in \arg \max_{s'_i} EU_i(s'_i, s_{-i}) = \arg \max_{s'_i} \sum_{\theta_i} p(\theta_i) EU_i(s'_i, s_{-i} | \theta_i)$$

for each i .



Bayesian (Nash) Equilibrium



- Explicitly models behavior in an uncertain environment
- Players choose strategies to maximize their payoffs in response to others accounting for:
 - strategic uncertainty about how others will play and
 - payoff uncertainty about the value to their actions.